TRINITY COLLEGE FOR WOMEN NAMAKKAL

Department of Mathematics

ALGEBRA

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Presented by

Dr.P.SARANYA

HOD-UG, Assistant Professor

Department of Mathematics

http://www.trinitycollegenkl.edu.in/

INTRODUCTION

Counting principle is a powerful tool for deriving certain theorems in algebra. The process of counting the elements in two different ways and then comparing the two, yields the desired conclusions. We define a conjugacy relation in a group to derive the class equation of a finite group. As an application of this equation, we prove that cauchy's Theorem which asserts the existence of an element of order p in G whenever $p \ o(G)$, for a given prime p. The number of distinct conjugate classes of Sn is obtained as the number of partitions of n.

Cartesian Product

The Cartesian product A x B of two sets A and B is the set{(a,b) : a ε A, and b ε B}.

Relation

A subset R of A x A is called a relation on A. If (a,b) ε R, we write a ~ b.

Relation

A relation ~ on a set A is said to be an equivalence relation on A if for all a,b,c in A,

a ~ a (reflexivity)

a ~ b implies b ~ a (symmetry)

a ~ b and b ~ c imply a ~ c (transitivity)

Equivalence Relation

If $\sim$ is an equivalence relation on A and $a \varepsilon A$, the equivalence class of a is the set

$$C(a) = \{ x : x \sim a \}$$

Theorem

Any two equivalence classes on a set are disjoint or identical

Theorem

If ~ is an equivalence relation on a set A, then A is the union of its distinct equivalence classes

Definiton

Let G be a group and a, b ε G. We say a is conjugate of b, if there exists an element g ε G such that a = g-1 bg. We write a ~ b.

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Lemma

In a group G, the conjugacy relation is an equivalence relation.

Proof :

Reflexivity : Since $a = e^{-1} ae$, where e is identity element of G, $a \sim a$.

Symmetry : If $a \sim b$, then there exists $x \varepsilon G$ such that $b = x^{-1} ax$.

Then $(x-1)^{-1}\, b\, x -1 = (x -1)^{-1} (x^{-1} ax)\, x -1$

i.e., $x\, b\, x^{-1} = x (x^{-1} ax)\, x -1$.

$= (x x^{-1})\, a\, (xx^{-1})$.

$= a$

Therefore, $a = x\, bx^{-1}$

Therefore $b \sim a$

Transitivity : Let a ~ b and b ~ c.

Then

$b = x^{-1} ax$, for some x Î G and $c = y^{-1} by$, for some y ε G.

$$c = y^{-1} (x^{-1} ax) y$$

$$= (y^{-1} x^{-1}) a (xy)$$

$$= (xy)^{-1} a (xy)$$

Therefore a ~ c

Therefore conjugacy is an equivalence relation.

Definition

If a belongs to the group G, the equivalence class of a under the conjugacy relation is C (a) = {x : x ~ a }. It is called the conjucate class of a. If G is a finite group, then the number of elements in C (a) is denoted by Ca.

Lemma

If a belongs to aroup G, then N(a) = { x ε G : xa = ax } is a subgroup of G and is called normaliser of a.

THANK YOU

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