TRINITY COLLEGE FOR WOMEN NAMAKKAL
Department of Mathematics

CALCULUS OF VARIATIONS AND INTEGRAL EQUATIONS
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TYPES OF KERNELS

(i) Symmetric Kernel

(ii) Separable or Degenerate Kernel

(ii) Transposed Kernel

(iv) Iterated Kernels

(v) Resolvent Kernel or Reciprocal Kernel

(i) Symmetric Kernel:

A Kernel $k(x,t)$ is symmetric (or complex symmetric or Hermition) if

$$k(x,t) = \overline{(k(x,t))}$$

Where bar denotes the complex conjugate. A real kernel $k(x,t)$ is symmetric if

$$k(x,t) = k(t,x)$$

Example:

Sin(x + t), e^xt, x^3 t^3+x^2 t^2+xt+1 are all symmetric kernels.

(ii) Separable or Degenerate Kernel

A kernel which is particularly useful in solving the Fredholm equation has the form

$$k(x,t) = \sum_{i=1}^{n} a_i(x)\, b_i(t)$$

Where n is finite and a_i, b_i are linearly independent sets of functions. Such a kernel is called separable or degenerate kernel.

Note:

A degenerate kernel has a finite number of characteristic values.

(iii) Transposed Kernel:

The kernel $k^T(x,t) = k(x,t)$ is called the transposed kernel of $k(x,t)$.

(iv) Iterated Kernels:

(a) Consider Fredholm integral equation of the second kind

$$u(x)=f(x)+ \lambda\int_a^b k(x,t)u(t)dt \quad \ldots\ldots\ldots(1)$$

Then, the iterated kernels $k_n(x,t)$, $n=1,2,3,\ldots\ldots$ are defined as follows

$$k_1(x,t)=k(x,t)$$

and $k_n(x,t)= \int_a^b k(x,s)k_{n-1}(s,t)\, ds$, $n = 2,3,\ldots\ldots$

(b) Consider Volterra integral equation of the second kind

$$u(x)=f(x)+ \lambda\int_a^x k(x,t)u(t)dt$$

Then, the iterated kernels $k_n(x,t)$, $n = 1,2,3,\ldots\ldots$ Are defined as follows

$$k_1(x,t)=k(x,t) \text{ and}$$

$$k_n(x,t)= \int_a^x k(x,s)k_{(n-1)}(s,t)\, ds,$$

$n = 2,3,\ldots..$

(v) Resolvent Kernel or Reciprocal Kernel:

Consider the integral equations

$$u(x)=f(x)+\lambda\int_a^b k(x,t)u(t)dt \quad \ldots\ldots\ldots\ldots (1)$$

and

$$u(x)=f(x)+\lambda\int_a^x k(x,t)u(t)dt \quad \ldots\ldots\ldots\ldots (2)$$

Let the solution of (1) and (2) be given by

$$u(x)=f(x)+\lambda\int_a^b R(x,t;\lambda)f(t)dt$$

and

$$u(x)=f(x)+\lambda\int_a^x \Gamma(x,t;\lambda)\, f(t)dt$$

Then, $R(x,t;\lambda)$ or $\Gamma(x,t;\lambda)$ is called the resolvent kernel or reciprocal kernel.

EIGEN VALUES AND EIGEN FUNCTION:

Consider the homogeneous Fredholm integral equation

$$u(x) = \lambda\int_a^b k(x,t)\ u(t)dt \quad \ldots\ldots\ldots\ldots (1)$$

Then values of the parameter λ for which (1) has a non-zero solution [(u(x) ≠0)] are called eigen values of (1) or of the kernel k(x,t), and every non-zero solution of (1) is called an eigen function corresponding to the eigen value λ.

Remarks:

(1)The eigen values are also known as characteristic values or characteristic numbers.

(2)Eigen functions are also known as characteristic function or fundamental functions.

(3)The number $\lambda=0$ is not an eigen value, since for $\lambda=0$, it follows from (1) that $u(x) = 0$.

(4)If $u(x)$ is an eigen function of (1), then $C.u(x)$, where C is an arbitrary constant, is also an eigen function of (1), which corresponds to the same eigen value λ.

THANK YOU

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