TRINITY COLLEGE FOR WOMEN NAMAKKAL

Department of Mathematics

CLASSICAL ALGEBRA

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MATRICES

Matrices: test for consistency of linear equation- characteristic equations- characteristic roots and characteristic vectors of a matrix-cayley-hamilton theorem-similarity of matrices-diagonalizable matrices-problems.

CHARACTERISTIC EQUATION OF A MATRIX

Let A be a n×n matrix over a field F and I be the unit matrix of the same order. Let I be the unknown. Then determinant |A_λI| is called the characteristic polynomial of the matrix A

The eqn |A_λI| =0 is called the characteristic eqn of the matrix.

The another name of characteristic root is latent root or eigen values .

CHARACTERISTIC VECTORS OF A [illegible]X:

*[illegible] A [illegible] n*n matrix. Let x be any non-zero column vector.*

$$X = \begin{bmatrix} x_1 \\ x_2 \\ \cdot \\ \cdot \\ \cdot \\ x_n \end{bmatrix}$$

They any solution of the equation $AX=\lambda X$ other the $X=0$ corresponding to some particular value of λ is called a characteristic vector or latent or eigen vector.

EXAMPLE:

Show that the two matrix A and 1/P(AP) have the same characteristic roots.

Solution.

let $B = 1/P(AP)$

$B-\lambda I = 1/P(AP) - \lambda I$

$= A - \lambda I$

$|B-\lambda I| = |A-\lambda I|$

$|B-\lambda I| = 0; \ |A-\lambda I| = 0$

The characteristic roots of A and B are the same.

CAYLEY-HAMILTON THEOREM

Every square matrix satisfies its own characteristic eqn.

ie, if the characteristic polynomiap is

$$\Phi(\lambda)=\lambda^{n}+P_{1}\lambda^{n-1}+P_{2}\lambda^{n-2} +P_{n-1}\lambda+P_{n}$$

$$\Phi(A)=0$$

$$A^{n}+P_{1}A^{n-2}+\ldots+P_{n-1}A+P_{n}I=0$$

DIAGONALIZABLE MATRIX

A matrix A is said to diagonalizable if it is similar to a diagonalizable matrix. Then there exist a non singular matrix ρ such that

$$\rho^{-1}A\rho=D$$

where D is a diagonal matrix

TEST FOR CONSISTENCY OF LINEAR EQUATION

WORKING RULE:

Write the given system of equation in the form AK=B.

Find the matrix [A,B] of the given system.

Find the rank of A and the rank of[A,B] by applying the elementary row operation only.

Don't use elementary column operation.

The following three cases gives the answer

1. If $\rho(A)=\rho[A,B]=$ no. of variables, then the system is consistent and has unique solution.

2. If $\rho(A)=\rho[A,B]<n$, then the system is consistent and has infinite number of solution.

3. If $\rho(A)\neq\rho[A,B]$, then the system is in consistent and there is no solution for the system of the equation.

RANK OF THE MATRIX:

- A process atleast minor of order r which doesn't vanish.
- The number of r is said to be the rank of the matrix A.
- Every minor of A of order r+1 and higher order vanish.
- The rank of matrix of A is denoted by $\rho(A)$.
- If I is the unit matrix of order n its rank is n.
- The rank of null matrix is taken as 0

MINOR OF THE MATRIX.

We know that every square matrix has a determinant. If A is m×n matrix then the determinant of every square sub matrix of A is called a minor of matrix A.

Consider the matrix

$$\begin{bmatrix} 1 & 3 & 5 & 4 \\ 6 & 1 & 2 & 7 \\ 7 & 5 & 4 & 3 \end{bmatrix}$$

This is the matrix of ...en

$$\begin{bmatrix} 1 & 3 & 5 \\ 6 & 1 & 2 \\ 7 & 5 & 4 \end{bmatrix}$$

is a square sub matrix of order 3.

Find is a square sub matrix of order 2.

$$\begin{bmatrix} 2 & 7 \\ 4 & 3 \end{bmatrix}$$

THANK YOU

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