



TRINITY COLLEGE FOR WOMEN NAMAKKAL

Department of Mathematics

GRAPH THEORY

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INTRODUCTION

In Mathematics, Graph Theory is the study of graphs, which are mathematical structures used to model pairwise relations between objects. A graph in this context is made up of vertices which are connected by edges.

Graph : A graph G consists of a pair $(V(G), X(G))$, where $V(G)$ is a non – empty finite set whose elements are called ***points or vertices*** and $X(G)$ is another set of unordered pairs of distinct elements of $V(G)$. The elements of $X(G)$ are called ***lines*** or ***edges*** of the graph.

If $x = \{u, v\} \in X$ then the line x is said to ***join*** of u and v . The points u and v are said to ***adjacent*** if $x = u v$. We say that the points u and the line x are incident with each other.

MULTI GRAPH

If more than one line joining two vertices are allowed then the resulting object is called a ***multi graph***. Lines joining the same points are called ***multiple lines***.

PSEUDO GRAPH

If an object contains multiple lines and loops then it is called a ***pseudo graph***.

COMPLETE GRAPH

A graph in which any two distinct points are adjacent is called a ***complete graph***.

NULL GRAPH

A graph G whose edge set is empty is called a ***null graph*** or a ***totally disconnected graph***.

LABELLED GRAPH

A graph G is called **labelled** if its p points are distinguished from one another by names such as v_1, v_2, \dots, v_p .

BIPARTITE GRAPH

A graph G is called a **bigraph** or **bipartite graph** if the vertex set V can be partitioned into two disjoint subsets V_1 and V_2 such that every line of G joins a point of V_1 to a point of V_2 . (V_1, V_2) is called a **bipartition** of G .

Theorem : 1.1

The sum of the degrees of the points of a graph G is twice the number of lines in G . i.e. $\sum d(v_i) = 2q$.

Proof: Every line of G is incident with two points.

\therefore Every line contributes two degrees. There are q lines in (p, q) graph.

$\therefore \sum_{i=1}^p d(v_i) = 2q = 2 \times (\text{number of lines in } G)$.

$p \ i=1$

Problem

Show that in any group of two or more people, there are always two with exactly the same number of friends inside the group.

Solution: Construct a graph G by taking the group of people as the set of points and joining two of them if they are friends.

Then $\text{deg } v = \text{number of friends of } v$.

To prove that at least two points of G have the same degree.

Let v_1, v_2, \dots, v_p be the points of G , where $p \geq 2$.

Clearly $0 \leq \deg v_i \leq p - 1$ for each i .

Suppose no two points of G have the same degree.

Then the degree of points v_1, v_2, \dots, v_p are $0, 1, 2, \dots, p - 1$ in some order. But, a point of degree $(p - 1)$ is joined to every other point of G .

Hence, no point can have degree zero. This is a contradiction to the fact that point of G has degree zero.

Thus, there exist two points of G with the same degree.

THANK YOU

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