TRINITY COLLEGE FOR WOMEN
NAMAKKAL
Department of Mathematics

LINEAR ALGEBRA
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Introduction to Linear Transformations

Function T that maps a vector space V into a vector space W:

$$T : V \xrightarrow{\text{mapping}} W, \quad V, W : \text{vector space}$$

V: the domain of T

W: the codomain of T

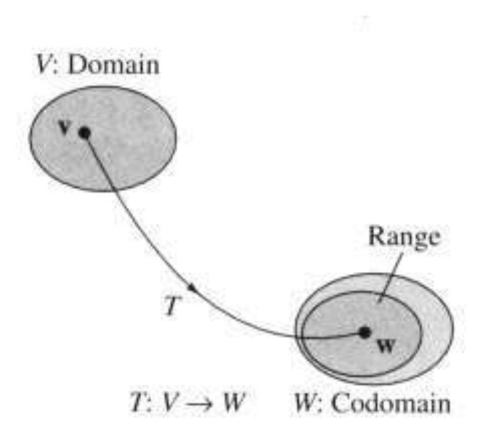

Image of $\mathbf{v}$ under T

If $\mathbf{v}$ is in V and $\mathbf{w}$ is in W such that

$$T(\mathbf{v})=\mathbf{w}$$

Then $\mathbf{w}$ is called the image of $\mathbf{v}$ under T .

Range of T

The set of all images of vectors in V.

Preimage of $\mathbf{w}$

The set of all $\mathbf{v}$ in V such that $T(\mathbf{v})=\mathbf{w}$.

Ex 1: (A function from R^2 into R^2)

$$T : R^2 \to R^2 \qquad \mathbf{v} = (v_1, v_2) \in R^2$$

$$T(v_1, v_2) = (v_1 - v_2, v_1 + 2v_2)$$

(a) Find the image of **v**=(-1,2). (b) Find the preimage of **w**=(-1,11)

Sol:

$$(a)\ \ \mathbf{v} = (-1, 2)$$

$$\Rightarrow T(\mathbf{v}) = T(-1, 2) = (-1 - 2, -1 + 2(2)) = (-3, 3)$$

$$(b)\ \ T(\mathbf{v}) = \mathbf{w} = (-1, 11)$$

$$T(v_1, v_2) = (v_1 - v_2, v_1 + 2v_2) = (-1, 11)$$

$$\Rightarrow v_1 - v_2 = -1$$

$$v_1 + 2v_2 = 11$$

$$\Rightarrow v_1 = 3,\ \ v_2 = 4$$

Thus {(3, 4)} is the preimage of **w**=(-1, 11).

Linear Transformation (L.T.):

V, W ： vector space

$T : V \rightarrow W$ ： V to W linear transformation

$$(1)\ \ T(\mathbf{u}+\mathbf{v}) = T(\mathbf{u}) + T(\mathbf{v}), \quad \forall \mathbf{u}, \mathbf{v} \in V$$

$$(2)\ \ T(c\mathbf{u}) = cT(\mathbf{u}), \quad \forall c \in R$$

(1) A linear transformation is said to be operation preserving.

$$T(\mathbf{u} \underset{\substack{\uparrow \\ \boxed{\text{Addition in } V}}}{+} \mathbf{v}) = T(\mathbf{u}) \underset{\substack{\uparrow \\ \boxed{\text{Addition in } W}}}{+} T(\mathbf{v}) \qquad T(c \underset{\substack{\uparrow \\ \boxed{\text{Scalar multiplication in } V}}}{\,} \mathbf{u}) = c \underset{\substack{\uparrow \\ \boxed{\text{Scalar multiplication in } W}}}{\,} T(\mathbf{u})$$

(2) A linear transformation $T : V \to V$ from a vector space into itself is called a **linear operator**.

Verifying a linear transformation T from R^2 into R^2

$$T(v_1, v_2) = (v_1 - v_2, v_1 + 2v_2)$$

PROOF

$$\mathbf{u} = (u_1, u_2),\ \ \mathbf{v} = (v_1, v_2) : \text{vector in } R^2,\ \ c : \text{any real number}$$

(1)Vector addition :

$$\mathbf{u} + \mathbf{v} = (u_1, u_2) + (v_1, v_2) = (u_1 + v_1, u_2 + v_2)$$

$$\begin{aligned} T(\mathbf{u} + \mathbf{v}) &= T(u_1 + v_1, u_2 + v_2) \\ &= ((u_1 + v_1) - (u_2 + v_2), (u_1 + v_1) + 2(u_2 + v_2)) \\ &= ((u_1 - u_2) + (v_1 - v_2), (u_1 + 2u_2) + (v_1 + 2v_2)) \\ &= (u_1 - u_2, u_1 + 2u_2) + (v_1 - v_2, v_1 + 2v_2) \\ &= T(\mathbf{u}) + T(\mathbf{v}) \end{aligned}$$

(2) Scalar multiplication

$$c\mathbf{u} = c(u_1, u_2) = (cu_1, cu_2)$$

$$\begin{aligned} T(c\mathbf{u}) &= T(cu_1, cu_2) = (cu_1 - cu_2, cu_1 + 2cu_2) \\ &= c(u_1 - u_2, u_1 + 2u_2) \\ &= cT(\mathbf{u}) \end{aligned}$$

Therefore, T is a linear transformation.

Two uses of the term "linear"

(1) $f(x)=x+1$ is called a linear function because its graph is a line.

(2) $f(x)=x+1$ is not a linear transformation from a vector space R into R because it preserves neither vector addition nor scalar multiplication.

THANK YOU

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