TRINITY COLLEGE FOR WOMEN
NAMAKKAL
Department of Mathematics

NUMERICAL ANALYSIS
21PMAE03-EVEN Semester

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PICARD'S METHOD OF SUCCESSIVE APPROXIMATIONS

consider the first order differential equations

$$\frac{dy}{dx}=f(x,y)$$

subject to y(xo)=yo

The equation can be written as

dy=f(x,y)dx

Intergating between the limits

$$\int_{yo}^{y} dy = \int_{x_0}^{x} f(x, y)\,\mathrm{dx}$$

$$\mathrm{Y} = y_0 + \int_{x_0}^{x} f(x, y)\,\mathrm{dx}$$

FIRST APPROXIMATIONS,

$$y_1 = y_0 + \int_{x_0}^{x} f(x, y_0)\,\mathrm{dx}$$

SECOND APPROXIMATIONS,

$$Y_2 = y_0 + \int_{x_0}^{x} f(x, y_0)\,\mathrm{dx}$$

nth approximations,

$$y_n = y_0 + \int_{x_0}^{x} f(x, y_{n-1})\,\mathrm{dx}$$

PICARD'S METHOD FOR SIMULTANEOUS FIRST ORDER DIFFERENTIAL EQUATIONS

Let $\frac{dy}{dx}$= f(x,y,z) and $\frac{dy}{dx}$ = $\varphi\ (x, y, z)$ be the simultaneous differential

Equation with intial conditions y(x0) and z(x0) = z_0

$$y_1 = y_0 + \int_{x_0}^{x} f(x, y_0, z_0)dx$$

$$z_1 = z_0 + \int_{x_0}^{x} \oint f(x, y_0, z_0)dx$$

PICARD'S METHOD FOR SIMULTANEOUS SECOND ORDER DIFFERENTIAL EQUATIONS

consider the second order differential equation

$$d^2y / d^2x = f(x, y, \frac{dy}{dx})$$

by $\frac{dy}{dx}$=z it can be reduced to two first order simulataneous differential equations

$$\frac{dy}{dx} = z \text{ and } \frac{dz}{dx} = f(x,y,z)$$

EULER'S METHOD

consider the differential equation

$$\frac{dy}{dx}=f(x,y)$$

where $y(x_0) = y_0$

To find successively $y_1, y_2, \ldots, y_m$. where y_m is the value of y corresponding to $x=x_m$, $x_m=x_0+mh, m=1,2,\ldots h$.

The equation of tangent at (x_0, y_0)

$$y=y_o+(x-x_0)f(x_0,y_o)$$

In general,

$$y_{m+1}=y_m+hf(\lambda_m,y_m)$$

IMPROVED EULER'S METHOD

The modified version of a well known variant of euler method known as the improved euler method

The quation of the coordinates $x1$,$y1$ is,

$$y_1 = y_0 + \frac{h}{2}\{f(x_0, y_0) + f(x_0 + h, y_0 + hf(x_0, y_0)\}$$

In we have the formula,

$$y_{m+1} = y_m + \frac{h}{2}\{f(x_m, y_m) + f(x_m + h, y_m\ hf(x_m, y_m)\}$$

MODIFIED EULERS METHOD

To improve the estimate of the slope ,determine two dervatives for the interval :

(1)At the initial point.
(2)At the end point.

The two dervitavies are then averaged to obtain an improved estimate of the slope for the entire interval.

$$y_1 = y_0 + (x - x_0)\{f x_0 + \frac{h}{2}, y_0 + \frac{h}{2} f(x_0, y_0)\}$$

$$=y_0+hf(x_0+\frac{h}{2}, y_0+\frac{h}{2}f(x_0,y_0)$$

preceeding in this way,

$$y_{m+1}=y_m+hf(x_m+\frac{h}{2}, y_m+\frac{h}{2}f(x_m,y_m).$$

THANK YOU

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