



# **TRINITY COLLEGE FOR WOMEN NAMAKKAL**

**Department of Mathematics**

**NUMERICAL ANALYSIS**

**21PMAE03 EVEN Semester**

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# RUNGE KUTTA METHOD

## INTRODUCTION:

- Developed by two german mathematicians Runge and kutta
- It is also called R-K Method.
  - These methods were developed around 1900.

# RUNGE'S KUTTA METHOD

Runge kutta method is an effective and widely used method for solving the initial value problems of differential equations.

Runge kutta method can be used to construct higher order accurate numerical method.

The algorithm to Runge Kutta method of second order is,

$$k_1 = hf(x_0, y_0) \quad , \quad k_2 = hf(x_0 + h, y_0 + k_1)$$

$$k = \frac{1}{2}(k_1 + k_2)$$

$$y_1 = y_0 + k.$$

## RUNGE KUTTA METHOD OF THIRD ORDER

FORMULAE;

$$K = \frac{1}{6}(K_1 + 4K_4 + K_3) \quad , \quad Y_1 = Y_0 + K$$

# RUNGE KUTTA METHOD OF FOURTH ORDER

$$K_1 = hf(x_0, y_0), \quad k_2 = hf\left(x_0 + \frac{h}{2}, y_0 + \frac{k_1}{2}\right)$$

$$k_3 = hf\left(x_0, \frac{h}{2}, y_0 + \frac{k_2}{2}\right),$$

$$k_4 = hf(x_0 + h, y_0 + k_3)$$

$$k = \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4)$$

$$y_1 = y_0 + k \quad \text{and} \quad x_1 = x_0 + h$$

Similar manner,

$$y_2 = y_1 + k \quad \text{and} \quad x_2 = x_1 + h$$

# RUNGE KUTTA METHOD FOR SIMULTANEOUS FIRST ORDER EQUATIONS

Consider the simultaneous  
equations ,

$$\frac{dy}{dx} = f_1(x, y, z); \frac{dz}{dx} = f_2(x, y, z)$$

If we consider the second order  
Runge kutta method then,

$$k_1 = hf_1(x_0, y_0, z_0), l_1 = hf_2(x_0, y_0, z_0)$$

$$k_2 = hf_1(x_0 + h, y_0 + k_1, z_0 + l_1);$$

$$l_2 = hf_2(x_0 + h, y_0 + k_1, z_0 + l_1)$$

$$k = \frac{1}{2}(k_1 + k_2)$$

$$l = \frac{1}{2}(l_1 + l_2),$$

$$y_1 = y_0 + k \quad \text{and} \quad z_1 = z_0 + l$$

# RUNGE KUTTA METHOD FOR SECOND ORDER DIFFERENTIAL EQUATIONS

Consider the second order  
differential equations

$$\frac{d^2y}{dx^2} = \varphi\left(x, y, \frac{dy}{dx}\right) \quad y(x_0) = y_0$$

$$\text{let } \frac{dy}{dx} = z \quad \text{then } \frac{d^2y}{dx^2} = \frac{dz}{dx}$$

$$\frac{dz}{dx} = \varphi(x, y, z), \quad z(x_0) = z_0$$



The problem reduce to sloving the simultaneous equation,

$$\frac{dz}{dx}=z=f_1(x,y,z) \text{ and } \frac{dz}{dx}=z=f_2(x,y.z)$$

subject to  $y(x_0)=y_0; z(x_0)=z_0$

and can be sloved as shown in the previous section.

# THANK YOU

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