TRINITY COLLEGE FOR WOMEN NAMAKKAL

Department of Mathematics

PARTIAL DIFFERENTIAL EQUATIONS(Part-I)

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Rules for finding Complementary Function:

Consider the equation

$$(D^2 + a_1DD' + a_2D'^2)\,Z = 0 \quad \text{-----------(1)}$$

where,

$$D = \frac{\partial}{\partial x} \quad \text{and } D' = \frac{\partial}{\partial y}$$

the auxilary equation is,

$$D'^2 \left[\left(\frac{D}{D'}\right)^2 + a_1\left(\frac{D}{D'}\right) + a_2 \right] Z = 0$$

$$\left[\left(\frac{D}{D'}\right)^2 + a_1\left(\frac{D}{D'}\right) + a_2 \right] Z = 0$$

Now, let D/D' = m

$$m^2 + a_1m + a_2 = 0 \quad \text{-------------(2)}$$

m_1 and m_2 be the two roots.

Case (i) : $m_1 \neq m_2$

The equation 1 can be written as

$(D - m_1D')(D-m_2D') Z = 0$ ------------ (3)

Now the solution of $(D - m_2D') Z = 0$ will also be a solution of equation 3

ie) $(D - m_2D') Z = 0$

$P - m_2q = 0$

which is of Lagrange's form and auxillary equation is

$$\frac{dx}{1} = \frac{dy}{-m_2} = \frac{dz}{0}$$

Taking first two factors,

$$\frac{dx}{1} = \frac{dy}{-m_2}$$

$-m_2\, dx = dy$

integrating,

$-m_2x = y + c_1$

$y + m_2x = c_1$

Taking 2nd and 3rd factors

$$\frac{dy}{-m_2} = \frac{dz}{0}$$

0 dy = -dz m_2

integrating,

$m_2 z = c_2$

$z = c_2$

$Z = f_2\,(y+m_2x)$ is a solution of $(D- m_2D')Z = 0$

Similarly,

Equation (3) will also be satisfied by the solution of

$(D-D'm_1)Z = 0$ by,

$Z = f_1(y+m_1x)$ where 'f_1' is another arbitrary function.

The complete solution of equation (1) is,

$$Z = f_1(y+m_1x) + f_2(y+m_2x)$$

Case (ii) $m_1 = m_2 = m$

Equation (1) can be written as

$(D - mD')(D - mD')\, Z = 0$

$(D - mD')^2\, Z = 0$ ---------- (4)

Let $(D - mD')\, Z = u$ ------------------ (5)

(4) => $(D-mD')\, u = 0$

It's solution is $u = f\,(y+mx)$ [since, case (i)]

$(D - mD')\, Z = u$ takes the form

$(D - mD')\, Z = f\,(y+mx)$

$P - mq = f\,(y+mx)$

=> $\frac{dx}{1} = \frac{dy}{-m} = \frac{dz}{f\,(y+mx)}$

Taking 1st two factors,

$\frac{dx}{1} = \frac{dy}{-m}$

$-mdx = dy$

Integrating,

$c_1 - mx = y$

$y + mx = c_1$ ------------ (6)

Taking 1st and 3rd factors

$$dx = \frac{dz}{f\,(y + mx)}$$

$f\,(y + mx)\,dx = dz$

$f\,(c_1)\,dx = dz$

Integrating,

$$x\,f\,(c_1) = Z + C_2$$

$$Z = x\,f\,(C_1) + C_2 \qquad [\text{ since, } C_2 \text{is a constant } \}$$

$$Z = x\,f\,(y + mx) + C_2$$

The complete solution of equation is,

$$Z = f_1(y + mx) + x\,f_2\,(y + mx)$$

Generalizing the results of case (i) and case (ii) :

(i) If the roots of auxillary equation are m_1, m_2, ... then

$$C.F = f_1 (y + m_1x) + f_2 (y + m_2x) + f_2 (y + m_3x) + ...$$

where f_1, f_2, f_3,..... are all arbitrary functions.

(ii) If two roots of Auxillary Equation are equal.

ie) $m_1 = m_2 = m$

Then,

$$C.F = f_1(y + mx) + xf_2(y+m_2x) + f_3(y + m_3x) + ...$$

where,

f_1, f_2, f_3,..... are all arbitrary function.

(iii) If three roots of Auxillary equation are equal

ie) $m_3 = m_2 = m_1$

Then,

C.F = $f_1(y + mx) + xf_2(y+m_2x) + x^2 f_3(y + m_3x) +$

$f_4 (y + mx)$ ....

where , f_1, f_2, f_3,..... are all arbitrary functions and so on.

Examples :

1) $D^3 - 3D^2D' + 4D'3 \Rightarrow m^3 - 3m^2 + 4 = 0$

$(m+1)(m^2 - 4m + 4) = 0$

$(m+1)(m-2)(m-2) = 0$

$m = -1, 2, 2$

2) $r + s - 2t \Rightarrow m^2 + m - 2 = 0$

$(m-1)(m+2) = 0$

$m = 1, -2$

3) $D^2 - 2DD' + D'^2 \Rightarrow m^2 - 2m + 1 = 0$

$(m-1)(m-1) = 0$

$m = 1, 1$

THANK YOU

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