TRINITY COLLEGE FOR WOMEN NAMAKKAL
Department of Mathematics

REAL ANALYSIS I

19PMA02-ODD Semester

Presented by
Mrs V.GOKILA
Assistant Professor
Department of Mathematics
http://www.trinitycollegenkl.edu.in/

DEFINITIONS

- **Finite Set:**

  A set contains only finite number of elements

  Eg: consider A= {1,2,3,4,5}

- **Countable Set:**

  Eg: {N,W,Z,Q}

- **Uncountable Set:**

  Eg: {R, Q^C } are uncountable sets

➢ One to One Mapping

If y∈B, $f^{-1}(y)$ is the set of all x∈A such that f(x)=y.

If for each y∈B, $f^{-1}(y)$ consists atmost one element of A, then f is said to be one to one mapping of A into B.

➢ Note:

If f is one to one from A to B, then provided that $f(x_1) \neq f(x_2)$ whenever $x_1 \neq x_2$, $x_1, x_2 \in A$

➢ Results:

For any set A we say,

(i) A is finite if $A \sim J_n$ for some n (the empty set is also considered to be finite).

(ii) A is infinite if A is not finite.

(iii) If A is countable if $A \sim J$.

(iv) A is uncountable if A is neither finite nor countable.

(v) A is atmost countable if A is finite (or) countable. Countable sets are sometimes called **Enumerable** (or) **Denumerable.**

Example for Countable set:

Let A be the set of all integer. Then A is countable. For consider the following arrangement of the sets A and J:

A: 0,1,-1,2,-2,3,-3,...

J: 1,2,3,4,5,6,7,8,...

The formula for a function f from J to A which sets up a 1-1 correspondence,

$$f(n) = \begin{cases} n/2 & n \text{ is even} \\ -(n-1/2) & n \text{ is odd} \end{cases}$$

METRIC SPACE

A set X, whose elements we shall call points, is said to be a metric space if with any two points p and q of X there is associated a real numbers d(p,q) called the **distance from p to q** such that

(a) $d(p,q) > 0$ if $p \neq q$; $d(p,q)=0$;

(b) $d(p,q) = d(p,q)$

(c) $d(p,q) \leq d(p,r)+d(r,q)$ for any $r \epsilon X$.

➢ Some Important Results:

(a) For any collection $\{G_\alpha\}$ of open sets, $\bigcup_\alpha G_\alpha$ is open.

(b) For any collection $\{F_\alpha\}$ of closed sets, $\bigcap_\alpha F_\alpha$ is closed.

(c) Every neighborhood is an open set.

(d) A set E is open iff its complement is closed.

(e) Every finite set is compact.

(f) Closed subsets of compact sets are compact.

COMPACT SET

➢Definition: Open Cover

By an open cover of a set E in metric space X, we mean a collection {Gα} of open subsets of X such that $E \subset \cup_\alpha G_\alpha$.

➢Definition: Compact

A set K of a metric space X is said to be compact if every open cover of K contains a finite sub cover.

(ie) If {Gα} is an open cover of K, then there are finitely many indices $\alpha_1, \ldots, \alpha_n$ such that $K \subset G_{\alpha_1} \cup G_{\alpha_2} \cup \ldots \cup G_{\alpha_n}$.

➢ Definition: Separated

Two subsets A and B of a metric space X are said to be seprated if both $A \cap \bar{B}$ and $\bar{A} \cap B$ are empty.

(ie) If no points of A lies in the closure of B and no points of B lies in the closure of A.

➢ Definition : Connected

A set $E \subset X$ is said to be connected if E is not a union of two non empty separated sets.

➢ Result:

(i) Separated sets are disjoint.

(ii) Disjoint sets need not be separated.

THANK YOU

http://www.trinitycollegenkl.edu.in/