TRINITY COLLEGE FOR WOMEN NAMAKKAL
Department of Mathematics

REAL ANALYSIS I

21PMA02-ODD Semester

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CONVERGENT SEQUENCE

- **Definition:**

A Sequence $\{p_n\}$ in a metric space X is said to be **Converge** if there is a point $p \in X$ with the following property:

For every $\varepsilon > 0$ there is an integer N such that $n \geq N$ implies that $d(p_n, p) < \varepsilon$. (Here d denotes the distance in X)

We also say that $\{p_n\}$ converges to p (or) that p is the limit of $\{p_n\}$ and we write

$$\lim_{n \to \infty} p_n = P$$

If $\{p_n\}$ does not converges, it is said to be **Diverge**

SUBSEQUENCE

- **Definition:**

Given a sequence $\{p_n\}$, consider a sequence $\{n_k\}$ of positive integers, such that $n_1<n_2<n_3<\dots$. Then the sequence $\{p_{n_i}\}$ is called a subsequence of $\{p_n\}$.

If $\{p_{n_i}\}$ converges, its limit is called a **Subsequential limit** of $\{p_n\}$.

It is clear that, $\{p_n\}$ converges to p if and only if every subsequence of $\{p_n\}$ converges to p.

➢ Results:

(a) If $\{p_n\}$ is a sequence in a compact metric space X, then some subsequence of $\{p_n\}$ converge to a point of X.

(b) The subsequential limits of a sequence $\{p_n\}$ in a metric space X form a closed subset of X.

(c) In any metric space X, every convergent sequence is a cauchy sequence.

(d) If X is a compact metric space and if $\{p_n\}$ is a cauchy sequence in X, then $\{p_n\}$ converges to some points of X.

COMPLETE

- **Definition:**

A metric space in which every cauchy sequence converges is said to be complete.

- **Results:**

(a) All compact metric spaces and all Euclidean space are complete.

(b) Every closed subset E of a complete metric space X is complete.

(c) A metric space which is not complete is the space of all rational numbers with $d(x,y)=|x-y|$.

Series

➢Definition:

Given a sequence $\{a_n\}$

$\sum_{n=p}^{q} a_n$ (p≤q) to denote the sum $a_p+a_{p+1}+\ldots+a_q$

We associates a sequence $\{s_n\}$ where

$$S_n = \sum_{k=1}^{n} a_k$$

For $\{s_n\}$ we use the expression $a_1+a_2+a_3+\ldots$

(or)

$$\sum_{n=1}^{\infty} a_n$$

This is called an **infinite series** and the numbers s_n are called **partial sums of the series**.

➢ Results:

(a) If $\sum a_n$ converges, then $\lim_{n\to\infty} a_n = 0$

(b) $\sum_{n=1}^{\infty} \frac{1}{n}$ diverges (or) Show that then harmonic series $1 + \frac{1}{2} + \frac{1}{3} + \cdots$ diverges.

(c) $\sum a_n$ converges if and only if for every ε>0, there is an integer N such that $\left| \sum_{k=n}^{m} a_k \right| \leq \varepsilon$ if m≥n≥N.

➢ Comparison Test:

(a) If $|a_n| \leq c_n$ for n ≥ N_0, where N_0 is some fixed integer, and if $\sum c_n$ converges, then $\sum a_n$ converges.

(b) If $a_n \geq d_n \geq 0$ for n ≥ N_0 and if $\sum d_n$ diverges, then $\sum a_n$ diverges.

THE NUMBER e

- **Results:**

(i) $e = \sum_{n=0}^{\infty} \frac{1}{n!}$

(ii) $\lim_{n\to\infty} \left(1 + \frac{1}{n}\right)^n = e$

(iii) e is irrational

(iv) For any sequence $\{c_n\}$ of positive numbers,

$$\lim_{n\to\infty} \sup \sqrt[n]{c_n} = \lim_{n\to\infty} \sup \frac{c_{n+1}}{c_n}$$

THANK YOU

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