TRINITY COLLEGE FOR WOMEN NAMAKKAL
Department of Mathematics

REAL ANALYSIS I
21PMA02-ODD Semester

Presented by
Mrs V. GOKILA
Assistant Professor
Department of Mathematics
http://www.trinitycollegenkl.edu.in/

CONTINUITY

➢ Definition: Limits of Functions

Let X and Y be metric spaces; suppose $E \subset X$, f maps E into Y, and p is a limit of E. we write $f(x) \to q$ as $x \to p$, or

$$\lim_{x \to p} f(x) = q$$

If there is a point $q \in Y$ with the following property:

For every $\varepsilon > 0$ there exists a $\delta > 0$ such that $d_Y(f(x), q) < \varepsilon$

for all points $x \in E$ for each

$$0 < d_X(x, p) < \delta$$

➢ Important Results:

(i) If f has a limit at p, this limit is **unique**.

(ii) Let X and Y be metric spaces; suppose E⊂ X, f maps E into Y, and p is a limit of E.

Then $\lim_{x \to p} f(x) = q$ iff $\lim_{x \to p} f(p_n) = q$

for every sequences $\{p_n\}$ in E such that $p_n \neq p$,

$$\lim_{x \to \infty} p_n = p$$

(iii) Suppose we have two complex functions f and g both defined on E. By f+g we mean the function which assigns to each points x of E, the number f(x) + g(x).

CONTINUOUS FUNCTIONS

Definition: Continuous

Suppose X and Y are metric spaces, $E \subset X$, $p \in E$, and f maps E into Y. Then f is said to be Continuous at p if for every $\varepsilon > 0$ there exists a $\delta > 0$ such that

$$d_Y(f(x), f(p)) < \varepsilon$$

for all points $x \in E$ for which

$$d_X(x, p) < \delta$$

If f is continuous at every at every point of E, then f is said to be **continuous on E**

➢ Results:

(i) If X and/ or Y are replaced by the real line, the complex plane, or by the some euclidean space R^k, the distances d_X , d_Y are of course replaced by absolute values (or) by appropriate norms.

(ii) A mapping f of a metric space X into a metric space Y is continuous on X iff f^{-1} (V) is open in X for every open set V in Y.

(iii) Let f and g be complex continuous functions on a metric space X, then f+g, fg and f/g are continuous on X.

➢ Example:

If $x_1, \ldots, x_k$ are the coordinates of the point X $\in R^k$, the functions ϕ_i defined by

$$\phi_i(x) = x_i \quad (x \in R^k)$$

are continuous on R^k, since the inequality

$$|\phi_i(x) - \phi_i(y)| \leq |x-y|$$

Shows that we may take $\delta=\varepsilon$.

The functions ϕ_i are sometimes called the **Coordinate Functions.**

➢ DEFINITIONS:

Continuity and Compactness:

A mapping f of a set E into R^k is said to be **Bounded** if there is a real number M such that $|f(x)| \leq M$ for all $x \in E$.

Uniformly Continuous :

Let f be a mapping of a metric space X into a metric space Y. We say that f is continuous on X if for every $\varepsilon > 0$ there exists $\delta > 0$ such that $d_X(f(p), f(q)) < \varepsilon$ for all p and q in X for which $d_X(p,q) < \delta$ (depends on ε).

➢ Result:

Every uniformly continuous function is Continuous.

Discontinuous :

If x is a point in the domain of the function f at which f is not continuous, we say that f is discontinuous at x, or that **f is discontinuity at x.**

If f is defined on an interval or on a segment, we have to define the right hand and the left hand limits of f at x, which we denoted by $f(x+)$ and $f(x-)$ respectively.

Monotonic Functions:

Let f be on (a,b). Then f is said to be **Monotonically increasing** on (a,b) if $a<x<y<b$ implies $f(x) \le f(y)$.

Let f be on (a,b). Then f is said to be **Monotonically decreasing** on (a,b) if $a<x<y<b$ implies $f(x) \ge f(y)$.

The class of monotonic functions consists of both the increasing and decreasing functions .

Monotonic functions have no discontinuities of the second kind.

THANK YOU

http://www.trinitycollegenkl.edu.in/