TRINITY COLLEGE FOR WOMEN NAMAKKAL

Department of Mathematics

TOPOLOGY

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Presented by

Mrs.P.Nandhini.,M.Sc.,M.Phil.,(SET)

Assistant Professor

Department of Mathematics

http://www.trinitycollegenkl.edu.in/

CONNECTED SPACE

Separation:

Let X be a topological space. A separation of X is a pair of U,V of disjoint non empty subsets of X whose union is X.

Connected:

The space X is said to be connected, if there does not exists a separation of X.

Disconnected:

If the space X has a separation then it is said to be disconnected.

Example 1:

Let X denote two point space in the indiscrete topology. Obviously, there is no separation of X.

So, X is connected.

Example 2:

Let Y denote the subspace $[-1,0) \cup (0,1]$ of the real line R . Each of the sets $[-1,0) \,\&\, (0,1)$ is non empty and open in Y(although not in R).

They form a separation of Y.

(or neither of these sets contains a limit point of the other)

It is disconnected.

Example3:

Let X be the subspace $[-1,1]$ of the real line. The sets $[-1,0]$ & $(0,1]$ are disjoint & non empty, but they do not form a separation of X, because the first set is not open in X.

It is connected.

Example4:

The Rationals Q are not connected.

Therefore, the only connected subspaces of Q are the one- point sets.

If Y is a subspace of Q contains two-points p & q ; one can choose an irrational number a lying between p & q and write Y as the union of the open sets $Y \cap (-\infty, a) \,\&\, Y \cap (a, \infty)$

Example 5:

Consider the following subsets of the plane R^2

$$X = \{x \times y / y = 0\} \cup \left\{x \times y / x > 0 \,\&\, y = \frac{1}{x}\right\}$$

Then X is not connected.

Since by they form a separation of X because neither contain a limit point of the other.

Lemma:

If the sets C and D form a separation of X, and if Y is a connected subspace of X then Y lies entirely within C or D.

Proof:

Let Y be a connected subspace of X

Since by C and D are both open in X

$\Rightarrow C \cap Y \ \& \ D \cap Y$ are open in Y

(Since Y is a subset of X)

Given that C and D form a separation of X.

Implies that C & D are disjoint nonempty open sets of X such that $X = C \cup D$

$\Rightarrow C \cap Y \;\&\; D \cap Y$ are disjoint open sets of Y and their union is Y

$$\Rightarrow Y = (C \cap Y) \cup (D \cap Y)$$

If $C \cap Y \ \& \ D \cap Y$ are non empty then they form a separation of Y

Which is not possible (Since by Y is connected)

Therefore, either $C \cap Y = \phi$ or $D \cap Y = \phi$

Therefore, Y lies in D or Y lies in C

Hence the Proof.

THANK YOU

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