TRINITY COLLEGE FOR WOMEN NAMAKKAL

TRIGNOMETRY AND ANALYTICAL GEOMETRY OF 3D

21UMA08 - EVEN SEMESTER

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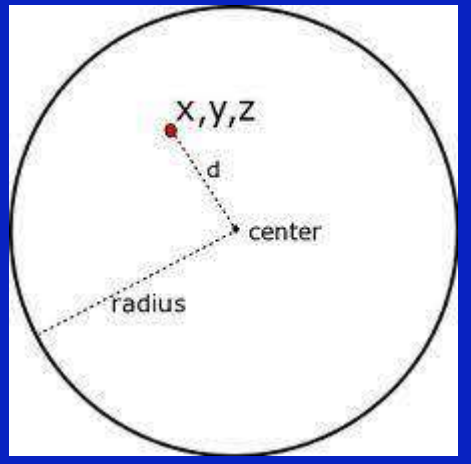
SPHERE

- DEFINITION
- EQUATION OF A SPHERE
- LENGTH EQUATION OF THE TANGENT
- THE PLANE SECTION OF A SPHERE IS A CIRCLE
- EQUATION OF A CIRCLE ON A SPHERE

DEFINITION :

A **sphere** is the locus of a point which moves in such a way that its distance from a fixed point is always CONSTANT. The fixed point is called the **Centre of the sphere** and the constant distance the **radius of the sphere .**



EQUATION OF A SPHERE :

When the center and radius are given ;

$$x^2+y^2+z^2-2ax-2by-2cz+(a^2+b^2+c^2-r^2) = 0$$

here r be the radius of the sphere and C (a,b,c) be the Centre of the sphere then P (x,y,z) be any point of a sphere .

EXAMPLE :

1) Find the equation of the sphere with center (2,3,1) and the radius 5 units .

Solution:

We know that the equation of a sphere whose centre is (a,b,c) and radius r is,

$$(x-a)^2 + (y-b)^2 + (z-c)^2 = r^2 \quad \text{---------- (1)}$$

Given : centre is (2, 3, 1) and radius is 5 units.

Here a=2, b=-3, c=1 and r=5 Substituting these values in equation (1)

we get $(x-2)^2 + (y-(-3))^2 + (z-1)^2 = 5^2$ $(x-2)^2 + (y+3)^2 + (z-1)^2 = 5^2$

ie., $x^2 + 4 - 4x + y^2 + 9 + 6y + z^2 + 1 - 2z = 25$

ie., $x^2 + y^2 + z^2 - 4x + 6y - 2z - 11 = 0$.

Which is the required equation of sphere

GENERAL FORM OF EQUATION OF THE SPHERE :

$$(x+u)^2 + (y+v)^2 + (z+w)^2 = \left(\sqrt{u^2 + v^2 + w^2 - d}\right)^2$$

This equation shows that the equation of sphere whose centre is

(-u , -v , -w) and the radius is r = $\sqrt{u^2 + v^2 + w^2 - d}$.

NOTE :

The equation of sphere centre is (0,0,0) then the radius is written by

$x^2+y^2+z^2 = r^2$.

EXAMPLE :

Find the centre and radius of the sphere x2 +y2 +z2 +2x-4y-6z+5=0.

Solution:

Given : x2 +y2 +z2 +2x-4y-6z+5=0

We know that the general equation of a sphere is,

x 2 +y2 +z2 +2ux+2vy+2wz+d=0.

Here 2u=2 | 2v=-4 | 2w=-6 | d=5

ie., u=1 | v=-2 | w=-3 |

∴ Centre : (-u,-v,-w)= (-1,2,3)

Radius = √ (u2 +v2 +w2 -d)

= √ ((-1)2 +22 +32 -5)

=√ (1+4+9-5)

=√ 9

Hence the centre of the given sphere is (-1,2,3) and radius is 3 units .

LENGTH EQUATION OF THE TANGENT :

We find the length of the tangent from the point (x,y,z) to the sphere is X^2+y^2+z^2+2ux+2vy+2wz+d = 0 .

Then the length of the tangent is of the form

$$PT = \sqrt{X1^2 + y1^2 + z1^2 + 2ux1 + 2vy1 + 2wz1 + d}$$

EXAMPLE :

Find the equation of the tangent plane to the sphere X^2+y^2+z^2-4x+2y-6z+5=0

Which is parallel to the plane 3x+2y-2z = 0 .

THE PLANE SECTION OF A SPHERE IS A CIRCLE :

GREAT CIRCLE :

- If the plane passing through the centre of the sphere is known as a **great circle .**
- In this case ,the radius of the circle is equal to the radius of the sphere

SMALL CIRCLE :

The plane section of plane not passing through the centre of the Sphere are called **small circle .**

EQUATION OF A CIRCLE ON A SPHERE :

The section of a sphere is a circle ,therefore the circle can be represented by two equations ,are being of a sphere and other of a plane

This equation $X^2+y^2+z^2+2ux+2vy+2wz+d = 0$, $lx+my+nz = P$ taken together represented a circle .

EXAMPLE :

Find the equation of the sphere having the circle, $X^2+y^2+z^2-2x+4y-6z+7=0$, $2x-y+2z=5$ for break circle .

EQUATION OF A SPHERE PASSING THROUGH A GIVEN CIRCLE :

The equation ,

$$X^2+y^2+z^2+2ux+2vy+2wz+d +k (lx+my+nz-p) = 0$$

In which k is any constant represents a sphere.moreover the equation is satisfied by the co-ordinates of any point which is common to the sphere which passes through the circle .

$$X^2+y^2+z^2+2ux+2vy+2wz+d = 0$$

$$lx+my+nz = P$$

EXAMPLE :

Find the equation of the sphere which passes through $X^2+y^2+z^2-2x-4y=0$, $X+2y+3z=8$ and touches the plane $4x+3y=25$.

THANK YOU !!!

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