TRINITY COLLEGE FOR WOMEN
NAMAKKAL
Department of Mathematics

TRIGONOMETRY AND ANALYTICAL GEOMETRY OF 3D

21UMA08-Even Semester

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CONE

DEFINITION

CONE :

Cone is a ruled surface generated by a moving straight line which passes through a fixed point and intersect a fixed curve.

The fixed point is called the vertex and the fixed curve is called the guiding curve.

Since a generator is a straight line ; it extends on either side infinitely as such a cone is a infinite surface being symmetrical about its vertex.

EQUATION OF A CONE :

There are two types of the equation of a cone .

- **Cone with its vertex and the origin O.**
- **Cone with its vertex at A (α,β,ɣ).**

EXAMPLE :

Find the equation of the cone whose vertex is the point (1,1,0) and whose guiding curve is $x^2+z^2 = 4$,y = 0.

Let $Q(x_1,y_1,z_1)$ be a point on the guiding curve $x_1^2+y_1^2+z_1^2 = 4$,y = 0.

Let A be a vertex and $p(x_0,y_0,z_0)$ any point on the generated AQ. Let Q divide AP in the ratio k:1-k then ,

$$x_1 = Kx_0+(1-k)\alpha$$

$$y_1 = ky_0+(1-k)\beta$$

$$z_1 = Kz_0+(1-k)\gamma$$

The point (1,1,0)

$$x_1 = Kx_0+(1-k)1$$

$$y_1 = ky_0+(1-k)1$$

$$z_1 = Kz_0+(1-k)0$$

$$x_1 = 1 + K(x_0-1) \quad \Rightarrow 1$$

$$y_1 = 1 + k(y_0-1) \quad \Rightarrow 2$$

$$z_1 = Kz_0 \quad \Rightarrow 3$$

Put y = 0 in $\Rightarrow$ 2

$$0 = 1 + k(y_0 -1)$$

$$-1/y_0-1 = k$$

Sub k value in x and z

$$x_1 = 1 + (-1/y_0-1)(x_0-1)$$

$$x_1 = y_0-1-x_0+1/y_0-1$$

$$x_1 = y_0 - x_0/y_0 - 1$$

$$z_1 = -z_0 / y_0 -1$$

Sub x and z values in guiding curve

$$x_1^2 + z_1^2 = 4$$

$$(y_0 - x_0/y_0 - 1)^2 + (- z_0/ y_0 - 1)^2 = 4$$

$$(y_0 - x_0)^2 + (z_0)^2 / (y_0 - 1)^2 = 4$$

$$(y_0 - x_0)^2 + (z_0)^2 = 4(y_0 - 1)^2$$

$$y_0^2 + x_0^2 - 2y_0x_0 + z_0^2 = 4(y_0 - 1)^2$$

$$y_0^2 + x_0^2 - 2y_0x_0 + z_0^2 = 4(y_0^2+1-2y_0)$$

$$x_0^2 +y_0^2 - 2y_0x_0 + z_0^2 - 4y_0^2 -4 + 8y_0 = 0$$

$$x_0^2- 3y_0^2+ z_0^2 - 2x_0y_0 +8y_0 -4 = 0$$

Replace x_0,y_0,z_0 be x,y,z

$$x^2 -3y^2 + z^2 - 2xy + 8y - 4 = 0$$

CONE WHOSE VERTEX IS AT THE ORIGIN

DEFINITION :

HOMOGENEOUS FUNCTION .

An equation $f(x,y,z) = 0$ is said to be homogeneous in (x,y,z) degree nit.

If $(tx,ty,tz) = t^n f(x,y,z)$ for all real values of t.

EXAMPLE :

Consider $ax^2+by^2+cz^2+2fyz+2gzx+2hyx= 0$ is a homogeneous equation.

$$f(x,y,z) = ax^2+by^2+cz^2+2fyz+2gzx+2hyx = 0$$

than

$$\text{if } (tx,ty,tz) = at^2x^2+bt^2y^2+ct^2z^2+2ftytz+2gtztx+2htxty = 0$$

$$t^2 (ax^2+by^2+cz^2+2fyz+2gzx+2hxy) = 0$$

$$t^2f(x,y,z) = 0$$

$$f(tx,ty,tz) = t^2f(x,y,z) = 0 \text{ for all real value of t.}$$

QUADRIC CONE WITH VERTEX AT THE ORIGIN .

- Quadric cone is a cone whose equation is second degree cone.
- If the vertex of a cone is at the origin than its equation of homogeneous.
- Hence the equation of a quadric cone with its vertex is of the form.

EXAMPLE : $\mathbf{ax^2+by^2+cz^2+2fyz+2gxz+2hxy}$

To show that the equation of a quadric cone passing through the x,y,z axis of the form fyz+gzx+hxy = 0.

Consider the second degree homogeneous equation of a cone.

$$ax^2+by^2+cz^2+2fyz+2gzx+2hyx = 0$$

Since the vertex of the cone is the origin.

Now the cone passes through the x axis.

The point is (1,0,0)

This point satisties the equation 1

$$a = 0$$

It the cone passes through the y and z axes.

$$b = 0 \text{ and } c = 0$$

Sub these value in 1 we have

$$2(fyz+gzx+hxy = 0$$

$$fyz+gzx+hxy = 0$$

Hence proved.

GENERAL QUADRIC CONE

DEFINITION :

QUADRIC CONE

- The quadric cone is the simplest **quadric ruled surface** , i.e., it is a surface of degree 2 that contains infinitely many lines.
- I fact, the vertex of the cone is at the origin to a point of the surface lies on yhe cone.

EXAMPLE

To show that if the quadric cone has $ax^2+by^2+cz^2+2fyz+2gxz+2hxy = 0$ has three mutually perpendicular generator then $a+b+c = 0$.

Consider the equation of the quadric cone is,

$$ax^2+by^2+cz^2+2fyz+2gxz+2hxy = 0 \qquad \Rightarrow 1$$

Let g_1, g_2, g_3 be the three mutually perpendicular generator of the cone.

Then, g□ is perpendicular to the plane formed by g_1 and g_2 is

$$lx + my + nz = 0 \qquad \Rightarrow 2$$

Now, g_1 and g_2 are the line of intersection of the cone and this plane

If λ, μ, γ are the direction ratio of g_1 or g_2 then we have ,

$$a\lambda^2 + b\mu^2 + c^2 + 2f\mu\gamma + 2g\gamma\lambda + 2h\lambda\mu = 0 \qquad \Rightarrow 3$$

And

$$l\lambda + m\mu + n\gamma = 0 \qquad \Rightarrow 4$$

Solve the two equation we obtain the equation in λ/γ.

ie , $(\lambda/\gamma)^2 (am^2+bl^2-2hlm) + 2(\lambda/\gamma)(cl^2+an^2-2gnl) + (bn^2+cm^2-2fmn) = 0$

Whose roots are λ_1/γ_1 and λ_2/γ_2.

where λ_1,μ_1,γ_1 and λ_2,μ_2,γ_2 are the direction ratios of g_1 and g_2.

Hence the product of the root is

$$\lambda_1/\gamma_1 \times \lambda_2/\gamma_2 = bn^2+cm^2-2fmn / am^2+bl^2-2hlm \qquad \Rightarrow 5$$

and

$$\mu_1/\gamma_1 \times \mu_2/\gamma_2 = cl^2+an^2-2gnl / am^2+bl^2-2hlm \qquad \Rightarrow 6$$

Now g_1 and g_2 are perpendicular to each other , if $\lambda_1\lambda_2+\mu_1\mu_2+\gamma_1\gamma_2 = 0$.

From 5 and 6

$$(bn^2+cm^2-2fmn) + (cl^2+an^2-2gnl) + (am^2+bl^2-2hlm) \quad \Rightarrow 7$$

But since l,m,n are the directions cosines of the generator of the cone .

$$al^2 + bm^2 + cn^2 + 2fmn + 2\,gnl + 2\,hlm = 0 \quad \Rightarrow 8$$

From 7 + 8

$$bn^2 + cm^2 + cl^2 + an^2 + am^2 + bl^2 + al^2 + bm^2 + cn^2 = 0$$

÷ by 8

$$\Rightarrow (a+b+c) (l^2+m^2+n^2) = 0$$

$$\Rightarrow a+b+c = 0$$

Hence the proof.

THANK YOU

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