TRINITY COLLEGE FOR WOMEN
NAMAKKAL
Department of Mathematics

LAPLACE TRANSFORMS & FOURIER SERIES
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Topic: Laplace Transforms And Inverse Laplace Transforms Basic Problems

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Definition: Laplace Transform

Laplace transform is the integral transform of the given derivative function with real variable t, to convert into a complex function with variable s.

For $t \geq 0$, let f(t) be given and assume the function satisfies certain conditions to be stated later on.

The Laplace transform of f(t), that is denoted by L{**f(t)} or F(s)** is defined by the Laplace transform formula:

$F(s) = \int_0^\infty f(t)\, e^{-st}\, dt.$

Definition: Inverse Laplace Transform

The inverse Laplace transform is the transformation of a laplace transform in to a function of time.

If $L[(f(t)] =$ F(s) then f(t) is the inverse laplace transform of F(s)

The inverse being written as

f(t) $= L^{-1}$ {F(s)}

1. **Find the Laplace Transform of $tsin2t$.**

Soln:

W.K.T $L[t(f(t)] = -\frac{d}{ds}(F(s))$

$$L[tsin2t] = -\frac{d}{ds}(L(sin2t))$$

$$= -\frac{d}{ds}(\frac{2}{s^2+2^2}) = \left[\frac{4s}{(s^2+4)^2}\right]$$

2. **Find $L\left(e^{-3t}\, sint\, cost\right)$**

Soln:

$$L(e^{-3t}\, sint\, cost) = L\left[\frac{e^{-3t}}{2} sin2t\right] = \frac{1}{2} L[sin2t]_{s\to s+3}$$

$$= \frac{1}{2}\left[\frac{1}{s^2+2^2}\right]_{s\to s+3} = \frac{1}{2}\left[\frac{1}{(s+3)^2+4}\right]$$

$$L(e^{-3t}\, sint\, cost) = \frac{1}{2}\left[\frac{1}{s^2+6s+13}\right]..$$

3. Find the Laplace transform of $tcosht$.

Soln:

W.K.T, $L[tf(t)] = -\frac{d}{ds}[F(s)]$

$$L[tcosht] = -\frac{d}{ds}[L(cosht)]$$

$$= -\frac{d}{ds}[\frac{s}{s^2-a^2}] \qquad = -[\frac{s^2-a^2-2s^2}{(s^2-a^2)^2}]$$

$$L[tcosht] = [\frac{s^2+a^2}{(s^2-a^2)^2}].$$

4. **Find the Laplace transform of $e^{-t}sin2t$.**

Soln:

$$\mathrm{L}[e^{-t}\sin 2t]_{s=s+1} = \left[\frac{2}{(s^2+4)}\right]_{s=s+1} = \frac{2}{5}$$

5. **Find the Laplace transform of $\frac{s}{(s+1)^2}$.**

Soln:

$$L[f(t)] = \frac{s}{(S+1)^2}$$

$$[f(t)] = L^{-1}(\frac{s}{(S+1)^2})$$

$$L^{-1}\left(\frac{s}{(S+1)^2}\right) = \frac{d}{dt}\left[L^{-1}(\frac{1}{(S+1)^2})\right] \qquad = \frac{d}{dt}\left[e^{-t}L^{-1}(\frac{1}{(S)^2})\right]$$

$$= \frac{d}{dt}[e^{-t}t] = e^{-t}(1-t)$$

6. **Find the Laplace transform of $\frac{t}{e^t}$.**

Soln:

$$te^{-t} = -\frac{d}{ds}[F(s)]$$
$$= -\frac{d}{ds}[L(e^{-t})]$$
$$= -\frac{d}{ds}\left(\frac{1}{S+1}\right)$$
$$= -\left(\frac{1}{(S+1)^2}\right)$$

7. **Using Laplace Transform evaluate** $\int_0^{\infty} t\, e^{-2t} \sin t\, dt$

Soln:

W.K.T $\int_0^{\infty} e^{-st} f(t)dt = L[f(t)]$

$$\int_0^{\infty} e^{-2t}\, tsint\, dt = L[t\, sint\,]_{s=2}$$
$$= -\frac{d}{ds}[L(tsint)]$$
$$= -\frac{d}{ds}\left[\frac{1}{s^2+1}\right]_{s=2}$$
$$= \left[\left[\frac{2s}{(s^2+1)^2}\right]_{s=2}\right] \quad = \frac{4}{25}$$

7. **Find** $\boldsymbol{L^{-1}(cot^{-1}(s))}$

Soln:

$$L^{-1}[f(s)] = -\frac{1}{t}L^{-1}[f'(s)]$$

$$\boldsymbol{L^{-1}(cot^{-1}(s)) = \frac{-1}{t}L^{-1}(\frac{d}{ds}(cot^{-1}(s))}$$

$$= -\frac{1}{t}L^{-1}\left[\frac{-1}{1+s^2}\right] \quad = \frac{1}{t}\sin t$$

8. **Find** $L^{-1}\left[\frac{1}{s^2+4s+4}\right]$

Soln:

$$L^{-1}\left[\frac{1}{s^2+4s+4}\right] = L^{-1}\left[\frac{1}{(s+2)^2}\right] = e^{-2t}L^{-1}[\frac{1}{(s)^2}] \quad = e^{-2t}t$$

7. **Find the inverse Laplace transform of $\frac{e^{-2s}}{s-3}$.**

Soln:

W.K.T $L^{-1}[e^{-as}f(s)] = f(t-a)U(t-a)$

$$L^{-1}\left[\frac{1}{s-3}\right] = e^{3t}$$

$$L^{-1}\left[\frac{e^{-2s}}{s-3}\right] = e^{3(t-2)}U(t-2)$$

8. Find $L^{-1}\left[\frac{1}{s^2+6s+13}\right]$

Soln:

$$L^{-1}\left[\frac{1}{s^2+6s+13}\right] = L^{-1}\left[\frac{1}{(s+3)^2+4}\right] = e^{-3t}L^{-1}\left(\frac{1}{s^2+4}\right)$$

$$= \frac{e^{-3t}}{2}\sin 2t$$

9. Find the inverse laplace transform of $\frac{1}{(S+1)(S+2)}$

Soln:

Let

$$\frac{1}{(S+1)(S+2)} = \frac{A}{S+1} + \frac{B}{S+2} \text{ ....(1)}$$

1 = A(S+1)+ B(S+2)

A = 1; B = -1

$$(1)\rightarrow \frac{1}{S+1} - \frac{1}{S+2}$$

$$= L^{-1}\left[\frac{1}{S+1}\right] - L^{-1}\left[\frac{1}{S+2}\right]$$

$$= (e^{-t} - e^{-2t})$$

THANK YOU

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